سكشن رياضة رقم ٦

٢١/٣ (عيد الأم)

fx-991ES Plus

Two way Power

" Elementary fn "

$\boxed{أ}$ الدالة اللوغاريتمية

$$f(z) = Ln(z) = Ln(r) + i(\theta \pm 2n\pi)$$

*** Evaluate**

$\boxed{1}$ $Ln(z) = Ln(\sqrt{2} + i\sqrt{6})$

sol

$$r = \sqrt{2+6} = \sqrt{8}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{6}}{\sqrt{2}}\right) = \frac{\pi}{3}$$

$$Ln(z) = Ln(\sqrt{8}) + i\left(\frac{\pi}{3} \pm 2n\pi\right)$$

$\boxed{2}$ $3^{i} = e^{Ln(3^{i})} = e^{i Ln(3)}$

$$= \cos(Ln(3)) + i \sin(Ln(3))$$

$\boxed{3}$ $e^{z} = 2$

$Z = Ln(2) = Ln(2 + i0)$

$= Ln(2) + i(\pm 2n\pi)$ $\qquad r = \sqrt{4}$

$\theta = \tan^{-1}\left(\frac{0}{2}\right) = 0$

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* show that $\frac{d}{dz}\left(Ln(z)\right) = \frac{1}{z}$

sol

$f(z) = Ln\, z = \underbrace{Ln(r)}_{i} + \underbrace{i\theta}_{v}$

$u_r = \frac{1}{r}$ $\qquad \frac{-1}{r}$ $\qquad u_\theta = 0$

$v_r = 0$ $\qquad \frac{1}{r}$ $\qquad v_\theta = 1$

$f_n \longrightarrow$ analytic

$f'(z) = (u_r + i v_r)\, e^{-i\theta} = \frac{1}{r}\, e^{-i\theta}$

$= \frac{1}{r e^{i\theta}}$ $\qquad$ where $r e^{i\theta} = z$

$f'(z) = \frac{1}{z}$ #

* show that $(1+i)^i = e^{-(\frac{\pi}{4} \pm 2n\pi)} \cdot e^{\frac{i}{2}\ln(2)}$

L.H.S $= (1+i)^i = e^{\ln(1+i)^i} = e^{i\ln(1+i)}$

$$= \cos(\ln(1+i)) + i\sin[\ln(1+i)]$$

$\rightarrow \ln(1+i)$ $\qquad r = \sqrt{2},\ \theta = \frac{\pi}{4}$

$$\ln(1+i) = \ln(\sqrt{2}) + i\left(\frac{\pi}{4} \pm 2n\pi\right)$$

$$= e^{i\left[\ln(\sqrt{2}) + i\left(\frac{\pi}{4} \pm 2n\pi\right)\right]}$$

$$= e^{i[\ln(\sqrt{2})} \cdot e^{i\left(\frac{\pi}{4} \pm 2n\pi\right)]} \qquad \sqrt{2} = 2^{\frac{1}{2}}$$

$$= e^{i\ln(2)^{\frac{1}{2}}} \cdot e^{i\left(\frac{\pi}{4} \pm 2n\pi\right)}$$

$$= e^{\frac{i}{2}\ln(2)} \cdot e^{-\left(\frac{\pi}{4} \pm 2n\pi\right)}$$

2 الدالة المثلثية :

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \quad , \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sinh z = \frac{e^{z} - e^{-z}}{2} \quad , \quad \cosh z = \frac{e^{z} + e^{-z}}{2}$$

$$\cosh^2 - \sinh^2 = 1$$

$$\sinh(A+B) = \sinh A \cosh B + \cosh A \sinh B$$

$$\cosh(A+B) = \cosh A \cosh B + \sinh A \sinh B$$

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1 show that: $|\sin z|^2 = \sin^2 x + \sinh^2 y$

sol

$$\sin(z) = \sin(x+iy)$$

$$= \sin(x)\cos(iy) + \cos(x)\sin(iy)$$

$$= \sin(x) * \cosh y + \cos(x)\sinh(y)$$

$$|\sin z|^2 = \sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y$$

$$|\sin z|^2 = \sin^2(x)\left[1+\sinh^2 y\right] + \left[1-\sin^2 x\right]\sinh^2(y)$$

$$= \sin^2 x + \sinh^2 y \quad \#$$

$\boxed{2}$ <u>solve</u> $\cosh z = \frac{1}{2}$

$$\cosh z = \frac{e^z + e^{-z}}{2} = \frac{1}{2} \Rightarrow e^z + e^{-z} = 1$$

$$e^z - 1 + e^{-z} = 0 \qquad * e^z$$

$$\underset{a}{e^{2z}} - \underset{b}{e^z} + \underset{c}{1} = 0$$

$$e^z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$e^z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$z_1 = \mathcal{L}n^{\frac{1}{2} + i\frac{\sqrt{3}}{2}}$$

$$r = 1, \quad \theta = \frac{\pi}{3}$$

$$e^z = \frac{1}{2} - i\frac{\sqrt{3}}{2}$$

$$z_2 = \mathcal{L}n^{\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)}$$

$$r = 1, \quad \theta = 2\pi - \frac{\pi}{3}$$

$\cancel{z = \ln(1) + i\left(\frac{\pi}{3} \pm 2n\pi\right)}$ $z_1 =$

$$Z_1 = Ln(1) + i\left(\frac{\pi}{3} \pm 2n\pi\right)$$

$$Z_2 = Ln(1) + i\left(2\pi - \frac{\pi}{3} \pm 2n\pi\right)$$

$\boxed{3}$ show that :- $\sinh^{(-1)} z = Ln(z + \sqrt{z^2+1})$

$$L.H.S = \sinh^{(-1)} z = w$$

$$z = \sinh w = \frac{e^{w} - e^{-w}}{2}$$

$$2z = e^{w} - e^{-w}$$

$$e^{w} - 2z - e^{-w} = 0 \qquad * e^{w}$$

$$e^{2w} - 2z e^{w} - 1 = 0$$

$$e^{w} = \frac{2z \pm \sqrt{4z^2 + 4}}{2} = z \pm \sqrt{z^2+1}$$

$$w = Ln\left(z \pm \sqrt{z^2+1}\right)$$

where $w = \sinh^{(-1)} z$ //

Fuzzy

* axiom of Complement:-

1. $c(0) = 1 \quad , \quad c(1) = 0$

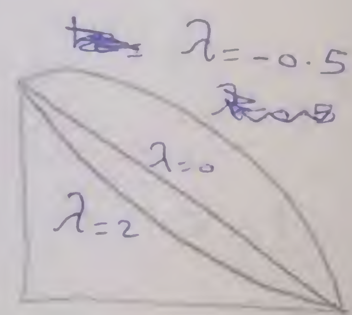
2. $a < b \longrightarrow c(a) > c(b)$

3. c is Continous fn.

4. $c(c(a)) = a$

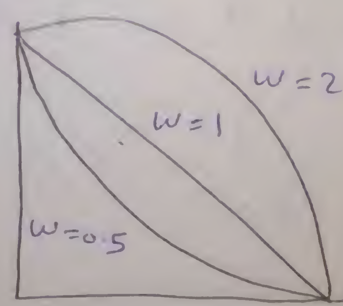
1. sugeno

$$c = \frac{1-a}{1+\lambda a} \qquad -1 < \lambda < \infty$$



2. Yager

$$c = (1-a^{w})^{\frac{1}{w}} \qquad 0 < w < \infty$$



* show that sugeno, Yager satisfied complement axioms.

solution

ــــ الدكتور قال يكفي انك تثبت أول شرطين .

→ Sugeno $C = \dfrac{1-a}{1+\lambda a}$

$\boxed{1}$ $c(0) = 1$, $c(1) = 0$

$\boxed{2}$ Let $a < b \Rightarrow -a > -b$

$1-a > 1-b \longrightarrow (1)$

$a\lambda < b\lambda \Rightarrow 1+a\lambda < 1+b\lambda$

$\dfrac{1}{1+a\lambda} > \dfrac{1}{1+b\lambda} \longrightarrow (2)$

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$\dfrac{1-a}{1+a\lambda} > \dfrac{1-b}{1+b\lambda}$

$c(a) > c(b)$ #

$\boxed{2}$ Yager $\quad C = (1 - a^{w})^{\frac{1}{w}}$

a) $c(0) = 1 \quad , \quad c(1) = (1-1)^{\frac{1}{w}} = 0$

b) $\cdot\ a < b \longrightarrow a^{w} < b^{w}$

$-a^{w} > -b^{w} \Longrightarrow 1 - a^{w} > 1 - b^{w}$

$(1-a^{w})^{\frac{1}{w}} > (1-b^{w})^{\frac{1}{w}} \Longrightarrow c(a) > c(b)$

$* \tilde{A} = \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.6}{3}$

$\boxed{1}$ Yager complement at $w=2$

$\boxed{2}$ α-cut at $\alpha = 0.4 \rightarrow w = 0.5, w = 1, w = 2$

$\boxed{\text{Sol}}$

$C = (1 - a^{2})^{\frac{1}{2}}$ نعوض في القانون بقيم (a) $\leftarrow$ 0.1, 0.2, 0.6

$\tilde{\bar{A}} = \frac{0.995}{1} + \frac{0.979}{2} + \frac{0.8}{3}$

$\boxed{9}$

at : $w = 2$

$$\tilde{A}_{0.4} = \frac{0.995}{1} + \frac{0.979}{2} + \frac{0.8}{3}$$

at $w = 0.5$ $\quad C = \left(1 - a^{0.5}\right)^2$

$$\cancel{\tilde{A} = \frac{0.468}{1} + \frac{0.31}{2} + \frac{0.05}{3}} \quad \tilde{A}_{0.4} = \{1, 2, 3\}$$

$$\tilde{A}_{0.4} = \{1\}$$

at $w = 1$ $\quad C = (1 - a)$

$$\tilde{A} = \frac{0.9}{1} + \frac{0.8}{2} + \frac{0.4}{3}$$

$$\tilde{A}_{0.4} = \{1, 2, 3\}$$

→ fuzzy unions (s-norms)

$A \cup B$

* axioms of union

$\boxed{1}$ $s(1,1) = 1$, $s(0,a) = a$

$\boxed{2}$ $s(a,b) = s(b,a)$

$\boxed{3}$ $a < a'$, $b < b' \Rightarrow s(a,b) < s(a',b')$

$\boxed{4}$ $s(s(a,b), c) = s(a, s(b,c))$

$\boxed{1}$ Domb

$$s(a,b) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}} \qquad 0<\lambda<\infty$$

$\boxed{2}$ Dubois Prade

$$s(a,b) = \frac{a+b-ab-\min(a,b,1-\alpha)}{\max(1-a, 1-b, \alpha)}$$

[3] Yager

$$s(a,b) = \min\left[1, (a^w + b^w)^{\frac{1}{w}}\right] \qquad 0 < w < \alpha$$

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* show Domb satisfied union axioms.

Sol

$s(1,1) = 1$, ~~also~~ ~~s(o,a)=~~

$$s(0,a) = \frac{1}{1+\left(\left(\frac{1}{a}-1\right)^{-\lambda}\right)^{-\frac{1}{\lambda}}} = \frac{1}{1+\frac{1}{a}-1} = a$$

$\longrightarrow s(a,b) = s(b,a)$

$$\frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}} = \frac{1}{1+\left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{a}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$\longrightarrow a < \grave{a}$

$$\frac{1}{a} > \frac{1}{\grave{a}} \Rightarrow \frac{1}{a}-1 > \frac{1}{\grave{a}}-1$$

$$\left(\frac{1}{a}-1\right)^{-\lambda} < \left(\frac{1}{a'}-1\right)^{-\lambda} \longrightarrow ①$$

$$b < b'$$

$$\therefore \left(\frac{1}{b}-1\right)^{-\lambda} < \left(\frac{1}{b'}-1\right)^{-\lambda} \longrightarrow ②$$

$$\left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{b}-1\right)^{-\lambda} < \left(\frac{1}{a'}-1\right)^{-\lambda} + \left(\frac{1}{b'}-1\right)^{-\lambda}$$

$$\left[\left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}} > \left[\left(\frac{1}{a'}-1\right)^{-\lambda} + \left(\frac{1}{b'}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\underbrace{1 + \qquad}_{A} > \underbrace{1 + \qquad}_{B}$$

$$\frac{1}{A} < \frac{1}{B}$$

$$\therefore s(a,b) < s(a',b')$$

$\boxed{4}$ $S(S(a,b),c) = S(a,S(b,c))$

$$L.H.S = S(S(a,b),c) = \frac{1}{1+\left[\left(\frac{1}{S(a,b)}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$S(a,b) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{-\frac{1}{\lambda}}}$$

$$\frac{1}{S(a,b)} = 1+\left[\underbrace{\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}}\right]^{\frac{-1}{\lambda}}$$

$$\frac{1}{S(a,b)}-1 =$$

$$\left(\frac{1}{S(a,b)}-1\right)^{-\lambda} = \left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}$$

$$S(S(a,b),c) = \frac{1}{1+\underbrace{\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}_{①}}$$

$\boxed{14}$ sec 6

$\underline{R.H.S} = s\left(a, s(b,c)\right)$

$$\cancel{\;} = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{s(b,c)}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$s(b,c) = \frac{1}{1+\left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$\frac{1}{s(b,c)} = 1+\left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\frac{1}{s(b,c)}-1 = \left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\left(\frac{1}{s(b,c)}-1\right)^{-\lambda} = \left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}$$

$$\boxed{s\left(a, s(b,c)\right) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}} \rightarrow (2)$$

$L.H.S = R.H.S$